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ACTIVE VIBRATION CONTROL OF A SMART BEAM

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ABSTRACT

This study presents an active vibration control technique applied to a smart beam. The smart beam consists of an aluminum beam modeled in cantilevered configuration with surface bonded piezoelectric (PZT) patches. The study uses ANSYS (v5.6) package program. The study first investigates the effects of element selection in finite element modeling. The effects of the piezoelectric patches on the resonance frequencies of the smart structure are also shown. The developed finite element model is reduced to a state-space form suitable for a controller design. The work then, by using this reduced model, presents the design of an active vibration controller which effectively suppresses the vibrations of the smart beam due to its first two flexural modes. The vibration suppression is achieved by the application of H_∞ controllers. The effectiveness of the technique in the modeling of the uncertainties is also presented.

INTRODUCTION

Utilization of discrete piezoelectric actuators have been shown to be a viable concept for vibration suppression in various works. Crawley and de Luis [1] proposed an analytical solution for a static case including various actuator geometries. They stated that discrete piezoelectric actuators could be considered in vibration suppression of some modes of vibration of flexible structures. Kalaycioglu and Misra [2] used a dynamic modeling technique for vibration suppression of plate structures by using PZT patches. The technique incorporates geometrical and mechanical properties of the actuator with the structures on which they mounted. Using the time-delay techniques, Kalaycioglu, Giray and Asmer [3] showed the effectiveness of the model on the active control of space structures. In one of the recent studies Suleman proposed the effectiveness of the piezoceramic sensor and actuators on the suppression of vibrations on an experimental wing due to gust loading [4].

The finite element method was shown to be a very effective tool for the analysis of the smart structures. The method offers fully coupled thermo-mechanical-electrical analysis of the structures, which makes simulation of highly interactive response of the system [5,6]. This allows the prediction of the reciprocal relations between the sensors and actuators and makes the development of the closed loop controller for active vibration control possible.

This study presents an active vibration control technique applied to a smart beam. The finite element model of the smart beam, which is composed of an aluminum cantilever beam and active PZT patches, is achieved by using ANSYS (v5.6) package program. Using this finite element model an H_∞ controller is designed which effectively suppresses the vibrations of the smart beam due to its first two modes. The effectiveness of the technique in the modeling of the uncertainties is also presented.

FINITE ELEMENT MODELING OF THE SMART BEAM

In the modeling and analysis of piezoelectric crystals the typical finite elements used are the solid elements, whereas in the analysis of thin plates, usually shell elements are utilized. The use of elements possessing different degrees of freedoms in the same model requires coupling of the consistent degrees of freedoms at the contact surfaces where these elements interface. Although the application of the coupling strategies guaranties the appropriate transfer of the nodal forces between the active and passive portions at the interface, the nodal moments corresponding to the nodal rotations do not transfer [7]. Hence, the first part of the study gives the effects of element selection in finite element modeling.

For this reason, a case given in reference [5] is considered. The smart beam was a 25×300×0.635 mm aluminum beam modeled in cantilevered configuration with single 25×63.5×0.19 mm PZT actuator placed on one surface of the beam closed to the clamped end.

In the current study, that smart beam was modeled again by considering two approaches. In the first approach the solid elements (SOLID5) were used for the modeling of the active portion (piezoelectric patches) and compatible solid elements (SOLID45) were used for the modeling of the passive portion (aluminum beam). This is called as 'hybrid solid-solid model'. Then the model given in the reference [5] is taken into the consideration and the passive structure was modeled with shell elements (SHELL99) whereas the piezoelectric patches were still solid elements. This second model was denoted as 'hybrid shell-solid model'. The specimen was then theoretically subjected to a piezoelectric actuation voltage of 400V. The mid-tip responses of the smart beam were theoretically calculated and the results were tabulated in Table 1 together with the experimental result given in reference [5]. Table 1 shows that the hybrid solid-solid model yields results which are closer to the experimental values. The differences between shell-solid and solid-solid hybrid models may be attributed to the improper modeling of the contribution of the element stiffness matrices to the global stiffness matrix of the beam. Those stem from the incompatibilities existing between the element types. Therefore, it can be concluded that in the use of the commercial code ANSYS, the hybrid models consisting of solid-solid elements do allow more precise modeling of the beams of this geometry.

Table 1. The comparison of the effects of the element type selection on the response

	Theoretical HYBRID SOLID-SOLID	Theoretical HYBRID SHELL-SOLID	Experimental Result [5]
Deflection(mm)	10.687	12.558	10

Figure 1 gives the geometry, dimensions and the finite element model of the smart beam used in this study. 8 (20×25×0.61 mm) Sensortech BM532 type actuators are glued in bimorph configuration on a 507×51×2 mm aluminum beam. The smart beam is modeled with hybrid solid-solid approach.

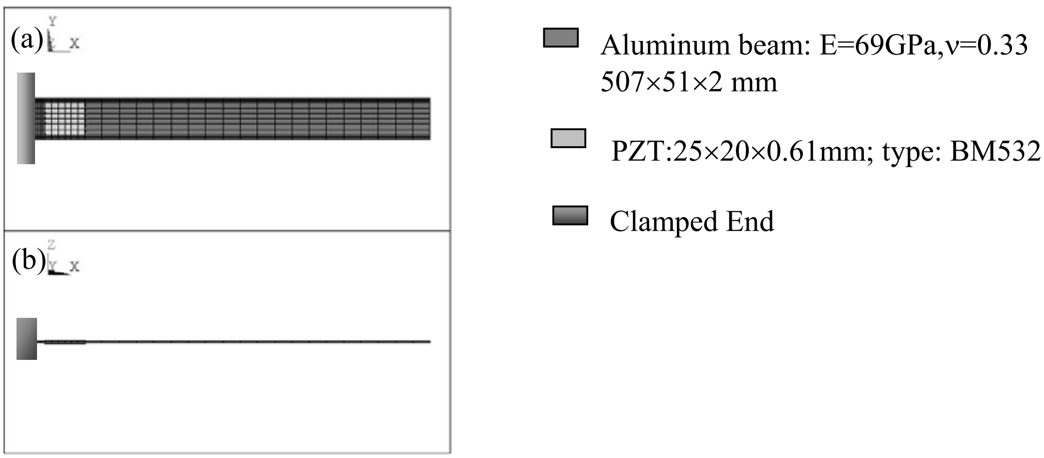


Figure 1. The geometry and the finite element model of the smart beam used in the study
(a)Top view
(b) Side view

The Influence of the Piezoelectric Patches on the Natural Frequencies

This section shows the effects of the eight piezoelectric patches on the natural frequencies of the aluminum beam. Table 2 compares the resonance frequencies of the passive aluminum and the smart beams. The presence of the actuators, as expected, shifts the natural frequencies to higher frequencies. This stiffening effects are more pronounced at low frequencies.

Table 2. The influence of the actuator patches on the resonance frequencies

Frequency(Hz)	Passive beam	Smart Beam	% Increase
f_1	6.679	7.503	%12.33
f_2	41.858	44.918	%7.31
f_3	117.219	121.06	%3.28

THE STATE SPACE REPRESENTATION OF THE SMART BEAM

The aim in the system modeling is to obtain the mathematical description of the plant suitable for the design of the control system. The system modeling technique includes the determination of the state space representation of the system. The model of the system can also be found via system identification [8,11]. The model obtained through system identification can also be used to tune the accuracy of the model derived from finite element method [8].

The Formulation of The State-Space Representation of the Smart Beam

The finite element method can effectively be used in the modeling of smart structures. In this study the nodal coordinates are selected in the modeling. The governing differential equation of motion for the smart beam can be represented as [9,10]

$$\left[M\right]\left\{\ddot{q}\right\}+\left[C\right]\left\{\dot{q}\right\}+\left[K\right]\left\{q\right\}=\left\{P\right\} \tag{1}$$

here, defining n as the degrees of freedom per node, M, C and K gives n×n mass damping and stiffness matrices respectively. The vector {q} represents the generalized vector of displacements, $\{\dot{q}\}$ gives the generalized vector of velocities, $\{\ddot{q}\}$ defines the vector of accelerations and{P} is the voltage to generalized force transformation vector.

In the modeling Rayleigh damping model is used. Rayleigh defined proportional damping as a dissipative situation where viscous damping matrix C is directly proportional to mass, stiffness or both as [7,10],

$$[C]=\gamma[M]+\beta[K] \tag{2}$$

Here, γ and β defines mass and stiffness material loss factors respectively. When the stiffness damping is used (γ=0) the modal loss factor ζ_r takes the form

$$\zeta_r=\frac{\omega_r}{2}\beta \tag{3}$$

In order to investigate the effectiveness of the controller at high frequencies very small material loss factor is considered in the theoretical calculations. In the study the material loss factor β is taken to be 1×10^{-4}.

In order to obtain a state space representation of the smart beam, the differential equation of motion described by equation (1) is premultiplied with M^{-1} (for nonsingular mass matrix)

$$\left\{\ddot{q}\right\}+M^{-1}D\left\{\dot{q}\right\}+M^{-1}K\left\{q\right\}=M^{-1}\left\{P\right\} \tag{4}$$

Furthermore, selection of the state vector x, as $\{q \quad \dot{q}\}^T$ leads to the formation of the specific form given in equation (5)

$$\begin{bmatrix} 0 & I \\ M^{-1}K & -M^{-1}D \end{bmatrix} \left\{ \dot{x} \right\} = \left\{ \begin{matrix} 0 \\ M^{-1}P \end{matrix} \right\} \tag{5}$$

where *I* defines n×n identity matrix. In this case, the output of the system can be written in the form of equation (6)

$$y = C_{oq}\left\{ q \right\} + C_{ov}\left\{ \dot{q} \right\} \tag{6}$$

Here, C_{oq} *and* C_{ov} give the displacement and velocity output vectors respectively. The forms of equation (5-6) allows the representation of the governing differential equation of motion given in equation (1), to be cast into the state space form as [9,10],

$$\left\{ \dot{x} \right\} = \left[A \right]\left\{ x \right\} + \left[B \right]\left\{ u \right\} , \qquad \left\{ y \right\} = \left[C_o \right]\left\{ x \right\} \tag{7}$$

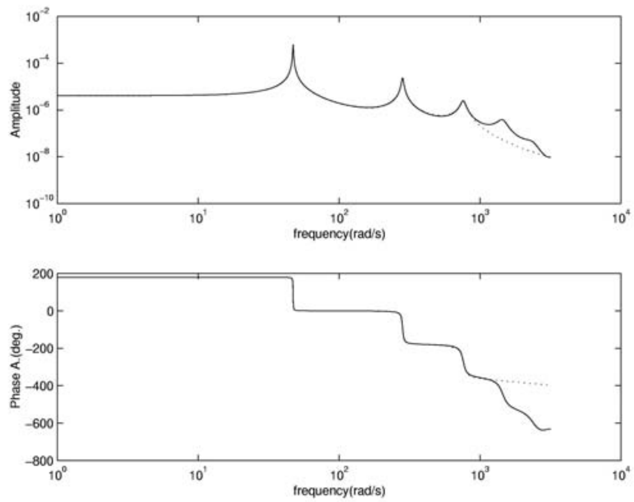
$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}P \end{bmatrix}, \quad C = \begin{bmatrix} Coq & C_{OV} \end{bmatrix} \tag{8}$$

The comparison of the equations (5-7) gives the form to be used in the controller design of the system consisting of *r* actuators and *s* sensors. Here, u is the $r \times 1$ input vector of the actuation voltage, A is $2n \times 2n$ system matrix, B $2n \times r$ input matrix and C_o $s \times 2n$ output matrix.

Model Reduction

In the finite element modeling, the structure is modeled to retain large number of degrees of freedoms for better accuracy. In active vibration control of flexible structures, however the use of smaller order model has computational advantages. Therefore, it is necessary to apply a model reduction technique to the state space representation. The reduced order system model extraction techniques solve the problem of the complexity by keeping the essential properties of the full model only [10-12]. The frequency range is selected to span first three frequencies of the smart beam in order to find the reduced order model of the system

During the theoretical calculations the 20th order system model obtained from the finite element model is reduced to the 6th order using a model reduction technique based on balance realization [12]. Figure 2 shows the comparison of the 6th and the 20th order system models



The 20^{th} order system
The 6^{th} order system

Figure 2. The Comparison of the 20^{th} and 6^{th} Order Models

H_∞ CONTROLLER DESIGN

This section gives the application of the H_∞ controller on the system model of the smart structure. The goal of the controller design is to increase the modal damping ratio within the frequency range of interest, therefore reducing the settling time. The effectiveness of the piezoelectric actuators in the open and closed loop controllers is shown in the literature [13-15]. In the H_∞ controller design aim is to minimize the H_∞ norm of the transfer function describing the relation between the inputs and the outputs of a multi input multi output system.

The H_∞ norm of the multi input multi output system **M** is defined as the supremum value of the singular values of the transfer function matrix calculated on the jω axis of the complex plane [11,13,15].

$$\|M\|_\infty = \sup \overline{\sigma}(M(j\omega)) \tag{9}$$

Here, *j* defines the complex number and $\overline{\sigma}(j\omega)$ is the largest singular value of the matrix $M(j\omega)$.

Closed loop architecture of the controller is shown in Figure 3. In this figure, *P(s)* defines the nominal transfer function of the system, *K* is the transfer function of the controller, *w, v* represents the noise signals and *z, e* symbolize the error signals. In this architecture, *K* controller processes the outputs and feeds back to the system. The H_∞ control problem consists of determining *K* such that the H_∞ norm of the transfer function from *w, v* to z, *e* is minimized and the closed loop system is stable.

Because of the measurement errors, the mismatches between the mathematical model and the time dependence of the parameters representing the system, no model can represent the real system exactly. In H_∞ controller design however, these uncertainities and errors can be included in the modeling systematically [11,12]. In this technique, the uncertainties are assumed to influence the linear time invariant system P(s) by means of another system *Δ(s)* as shown in Figure 4 In this configuration, despite the presence of the uncertainties *Δ*, the controller *K* minimizes the ratio of the signal energies *e* to *v*.

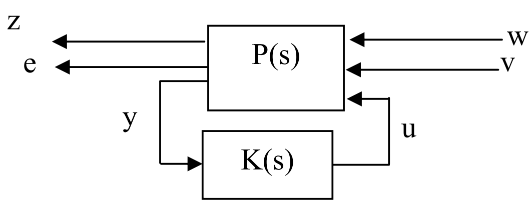


Figure 3. The closed loop Architecture of the H_∞ controller

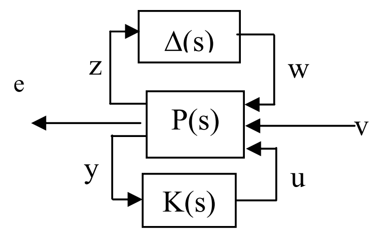


Figure 4. The Modeling of the Uncertainties

Figure 5 shows the formulation of the closed loop control problem in H_∞ framework. In this figure, SYS_{beam} defines the nominal smart beam model, δ is a complex number such that $\|\delta\| \langle 1$ and W_{add} defines the amplitude of the weight of additive uncertainty weight included into the system model.

The additive uncertainty weight W_{add} is included to account for the unmodeled or truncated high frequency modes. The interaction of the nominal transfer function SYS_{beam}, with *Δ* which is the multiplication of W_{add} by δ defines the system model of the smart beam including the uncertainties. In the modeling, W_{per} gives the performance weight applied to the displacement measurements made on the mid-tip of the smart beam.

In the study, W_d the weight added to disturbance, is taken to be 1 indicating that the order of the disturbance acting on the system and the input signal produced by the controller is the same. Furthermore, W_{noise}, representing the noise to signal ratio is selected to be 0.01

The goal in the controller design is to minimize displacement signal in the low frequency range, while not exciting the unmodeled high frequency modes [14]. Figure 6 gives the comparison of the frequency response of the beam and W_{add} .It can be seen from the figure that, as frequency increases the uncertainties increase indicating better system model at low frequencies. The comparison of W_{per} and the frequency response of the smart beam are shown in Figure 7. The application of this weight results in the minimization of the displacement at low frequencies while making minimal changes at high frequencies. W_{act} represents the weight applied to the actuator signals in order to limit the actuator authority. The weight is chosen as 0.01.

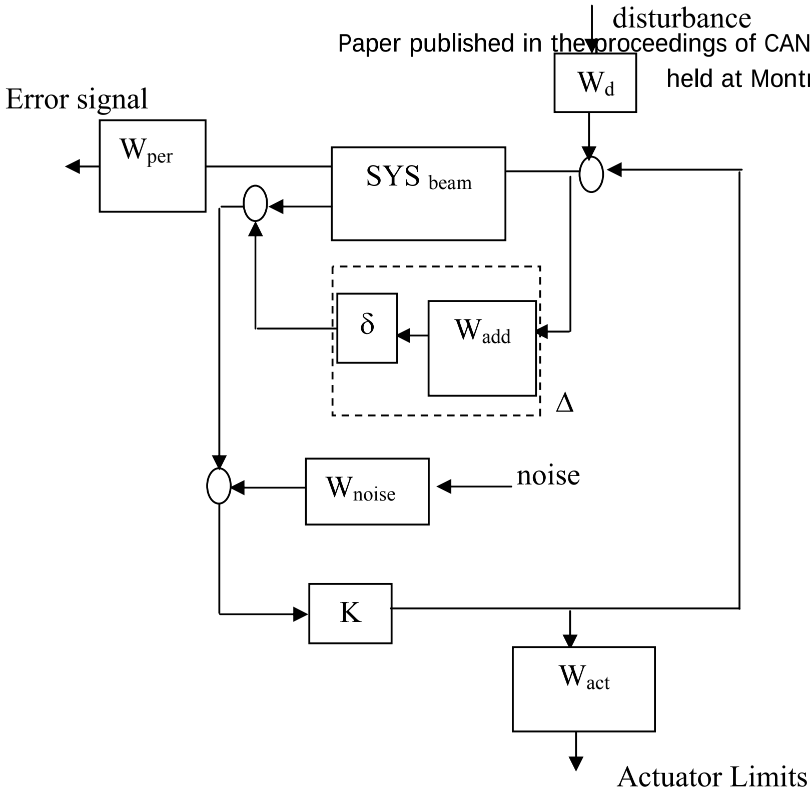


Figure 5. The Control Problem Formulation

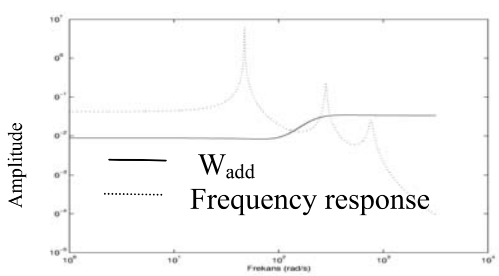


Figure 6. The Comparison of the W_{add} and the Frequency Response of the Smart Beam

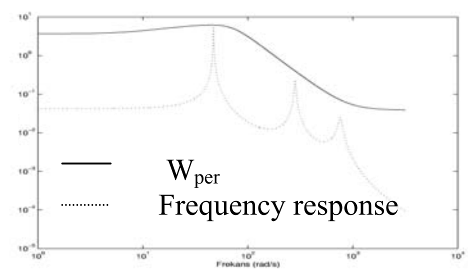


Figure 7. The Comparison of the W_{per} and the Frequency Response of the Smart Beam

The application of standard solution techniques to this problem leads to the determination of a 11th order controller. Figure 8 shows bode plot for this controller. The controller is then, reduced to the 4th order .The application of the controller to the system model results in 1/30 1/2.2 reduction at the amplitude of the first and second frequencies respectively. The comparison of the open and closed loop frequency response of the system are shown in Figure 9.

In order to test the robustness of the controller the structural singular value (μ) of the system is calculated across the frequency range of interest. For a given uncertainty structure Δ and closed loop system M, μ is defined as

$$\mu_{\Delta}(M) = \frac{1}{\min\{\overline{\sigma}(\Delta) : \Delta \in \Delta, \det(I - M\Delta) = 0\}} \qquad (10)$$

If no $\Delta \in \Delta$ makes $(I - M\Delta)$ singular then $\mu_{\Delta}(M) = 0$

A closed loop system will have robust performance (i.e. performance specification is satisfied by the closed-loop system in the presence of defined uncertainities) if μ less than 1 within the frequency range of interest. Figure 10 shows that the closed loop system designed for the smart beam has robust performance property [14,15]

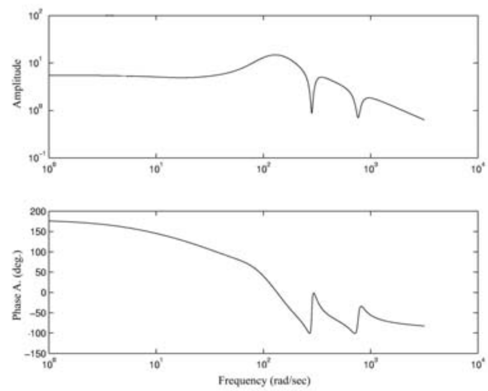


Figure 8. The 11th order controller Designed in the Study

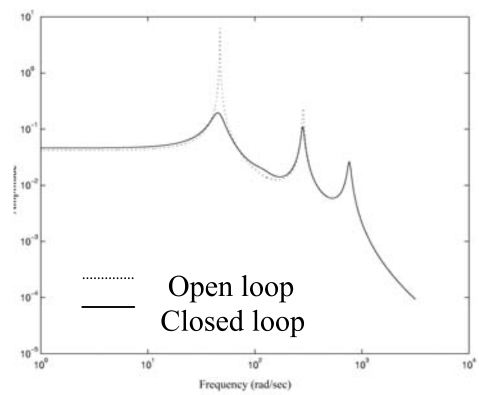


Figure 9.The comparison of the Closed and Open Loop Responses of the Smart Beam

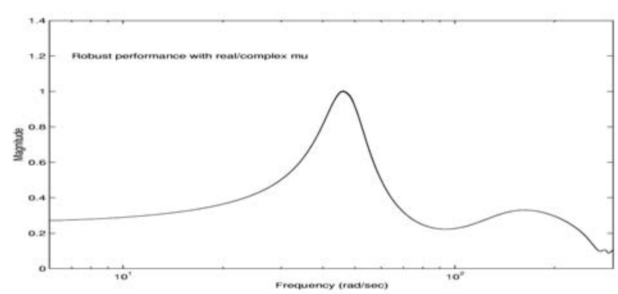


Figure 10. The structural Singular value (μ) of the Closed Loop System

DISCUSSION

The effects of the finite element type selection on the static response of the smart beam were shown. A finite element based modeling technique for the determination of the system model of the smart beam was presented. Based on this model, an H_∞ controller was designed which effectively suppresses the vibrations of the smart beam due to its first two modes. The suitability of the H_∞ design technique in the modeling of uncertainties and in evaluating the robust performance of the system was demonstrated.

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