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**APPLICATION OF SPATIAL H_∞ CONTROL TECHNIQUE
FOR ACTIVE VIBRATION CONTROL OF A SMART BEAM**

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ABSTRACT

This study presents the design and implementation of a spatial H_∞ controller for the active vibration control of a cantilevered smart beam. The smart beam consists of a passive aluminum beam (507x51x2mm) and eight symmetrically surface bonded SensorTech BM500 type PZT (Lead-Zirconate-Titanate) patches (25x20x0.5mm). PZT patches are used as actuators and a laser displacement sensor (Keyence LB-1201(W) LB-300) is used as sensor.

The smart beam was analytically modelled by using the assumed-modes method. The model only included the first two flexural vibrational modes and the model correction technique was applied to compensate the possible error due to the out of range modes. The system model was also experimentally identified and both theoretical and experimental models were used together in order to determine the modal damping ratios of the smart beam. A spatial H_∞ controller was designed for the suppression of the vibrations of the smart beam due to its first two flexural modes. The designed controller was then implemented and the suppression of the vibrations was experimentally demonstrated. This study also compared the effectiveness of a pointwise controller with the new spatial one.

Keywords: Assumed-Modes, Model Correction, Smart Beam, Spatial H_∞ Controller Design.

INTRODUCTION

The vibration is an important phenomenon for the lightweight flexible aerospace structures. That kind of structures may be damaged under any undesired vibrational load. Hence, they require a proper control mechanism to attenuate the vibration levels in order to preserve the structural consistency. The usage of smart materials, as actuators and/or sensors, has become promising research and application area that gives the opportunity to accomplish the reduction of vibration of flexible structures and proves to be an effective active control mechanism.

The smart structure is a structure that can sense external disturbance and respond to that with active control in real time to maintain mission requirements [1]. Active vibration control of a smart structure requires an accurate system model of the structure. Smart structures can be modeled by using analytical methods or system identification techniques using the experimental data [2, 3]. The system model of a smart structure generally involves a large number of vibrational modes. However, the performance goals are mostly related to the first few vibrational modes since their effect on structural failure is more prominent. Hence, a reduction of the order of the model is required [4, 5]. On the other hand, ignoring the higher modes can affect the system behavior since removing the higher modes directly from the system model perturbs the zeros of the system [6]. Therefore, in order to minimize the model reduction error, a correction term, including some of the removed modes, should be added to the model [6, 7].

Today, robust stabilizing controllers designed in respect of H_∞ control technique are widely used on active vibration control of smart structures. Yaman et al. [8, 9] showed the effect of H_∞ controller on suppressing the vibrations of a smart beam due to its first two flexural modes. Similar work is done for active vibration control of a smart plate, and usage of piezoelectric actuators on vibration suppression with H_∞ controller is successfully presented [10]. Ulker [11] showed that, beside the H_∞ control technique, μ -synthesis based controllers can also be successfully used to suppress the vibrations of smart structures.

Whichever controller design technique is applied, the suppression should be preferred to be achieved over the entire structure rather than at specific points, since the flexible structures are usually distributed parameter systems. Moheimani and Fu [12] and Moheimani et al. [13] introduced spatial H_2 and H_∞ norm concepts in order to meet the need of spatial vibration control, and simulation-based results of spatial vibration control of a cantilevered beam were presented. Moheimani et al. [14] studied spatial feedforward and feedback controller design, and presented illustrative results. They also showed that spatial H_∞ controllers could be obtained from standard H_∞ controller design techniques. Halim [15, 16] studied the implementation of spatial H_2 and H_∞ controllers on active vibration control and presented quite successful results. However these works were limited to a beam with simply supported boundary condition.

This paper aims to present design and implementation of a spatial H_∞ controller on active vibration control of a cantilevered smart beam.

THE SMART BEAM MODEL

The cantilevered smart beam model and its structural properties are given in Fig. 1 and Table 1, respectively. The smart beam consists of a passive aluminum beam (507mmx51mmx2mm) with symmetrically surface bonded eight SensorTech BM500 type PZT (Lead-Zirconate-Titanate) patches (25mmx20mmx0.5mm). The beginning and end locations of the PZT patches along the length of the beam are denoted as r_1 and r_2 , respectively. The patches are accepted as optimally placed [1]. The subscripts b and p indicate the beam and PZT patches, respectively. Note that, despite the actual length of the beam is 507mm, the effective length (i.e. the span of the beam) reduces to 494mm since it is clamped in the fixture.

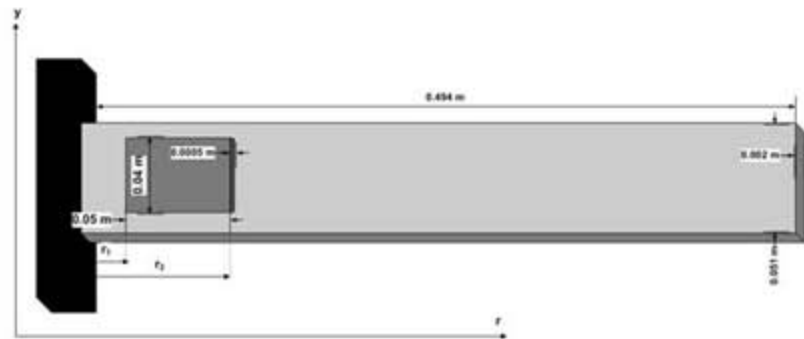


Fig. 1. The smart beam model used in the study

Table 1. Properties of the smart beam

	Aluminum Passive Beam	PZT
Length	$L_b = 0.494m$	$L_p = 0.05m$
Width	$w_b = 0.051m$	$w_p = 0.04m$
Thickness	$t_b = 0.002m$	$t_p = 0.0005m$
Density	$\rho_b = 2710kg / m^3$	$\rho_p = 7650kg / m^3$
Young's Modulus	$E_b = 69GPa$	$E_p = 64.52GPa$
Cross-sectional Area	$A_b = 1.02 \times 10^{-4} m^2$	$A_p = 0.2 \times 10^{-4} m^2$
Second Moment of Area	$I_b = 3.4 \times 10^{-11} m^4$	$I_p = 6.33 \times 10^{-11} m^4$
Piezoelectric charge constant	-	$d_{31} = -175 \times 10^{-12} m/V$

The assumed-modes model of the smart beam includes large number of resonant modes [17]. However, the control design criterion of this study is to suppress only the first two flexural modes of the smart beam. Hence, that high order model is directly truncated to a

lower order one, including only the first two flexural modes. The direct model truncation may cause the zeros of the system to perturb, which consequently affect the closed-loop performance and stability of the system considered [6]. For this reason, a general correction term k_i^{opt} is added to the truncated model and the resultant model [17, 19] can be expressed as:

$$\bar{G}_c(s, r) = \sum_{i=1}^2 \frac{\bar{P}_i \phi_i(r)}{s^2 + 2\xi_i \omega_i s + \omega_i^2} + \sum_{i=3}^{50} \phi_i(r) k_i^{opt} \quad (1)$$

where general correction constant is [18]:

$$k_i^{opt} = \frac{1}{4\omega_c \omega_i} \frac{1}{\sqrt{1-\xi_i^2}} \ln \left\{ \frac{\omega_c^2 + 2\omega_c \omega_i \sqrt{1-\xi_i^2} + \omega_i^2}{\omega_c^2 - 2\omega_c \omega_i \sqrt{1-\xi_i^2} + \omega_i^2} \right\} \bar{P}_i \quad (2)$$

and

$$\bar{P}_i = \frac{C_p [\phi_i'(r_2) - \phi_i'(r_1)]}{\rho_b A_b L_b^3 + 2\rho_p A_p L_p^3} \quad (3)$$

The nominal system model of the smart beam is denoted by $\bar{G}_c(s, r)$. The geometric constant $C_p = E_p d_{31} w_p (t_p + t_b)$ is due the bending moment of PZT patches exerted on the beam. The parameter r is the longitudinal axis and t is time. The cut-off frequency of the correction term is denoted by ω_c and the details of all the parameters and the detailed derivation of the equation (1) can be found in reference [19].

Theoretical assumed-modes modeling does not provide any information about the damping of the system. Experimental system identification, on the other hand, when used in collaboration with the analytical model, helps one to obtain more accurate spatial characteristics of the structure. The modal damping ratios and more accurate resonance frequencies were determined by spatial system identification [19] and the results are given in Table-2.

Table 2: The resonance frequencies and modal damping ratios of the smart beam

ω_1 (Hz)	ω_2 (Hz)	ξ_1	ξ_2
6.742	41.308	0.027	0.008

SPATIAL H_∞ CONTROL OF THE SMART BEAM

Controller Design:

Consider the closed loop system of the smart beam shown in Fig. 2. The aim of the controller, K , is to reduce the effect of disturbance signal over the entire beam by the help of the PZT actuators.

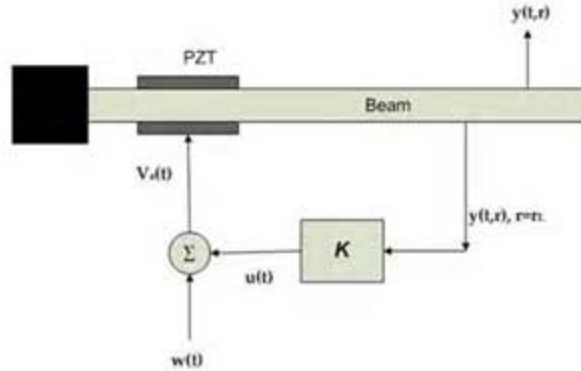


Fig. 2: The closed loop system of the smart beam

The state space representation of the system above can be shown to be [19]:

$$\begin{aligned}
 \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\
 y(t, r) &= C_1(r)x(t) + D_1(r)w(t) + D_2(r)u(t) \\
 \tilde{y}(t, r_L) &= C_2 x(t) + D_3 w(t) + D_4 u(t)
 \end{aligned} \tag{4}$$

where x is the state vector, w is the disturbance input, u is the control input, $y(t, r)$ is the performance output, $\tilde{y}(t, r_L)$ is the measured output at location $r_L = 0.99L_b$. The performance output represents the displacement of the smart beam along its entire body, and the measured output represents the displacement of the smart beam at a specific location, A is the state matrix, B_1 and B_2 are the input matrices from disturbance and control actuators respectively, Π is the output matrix of error signals, C_2 is the output matrix of sensor signals, Θ_1 , Θ_2 , D_3 and D_4 are the correction terms from disturbance actuator to error signal, control actuator to error signal, disturbance actuator to feedback sensor and control actuator to feedback sensor, respectively. The disturbance $w(t)$ is accepted to enter to the system through the actuator channels, hence, $B_1 = B_2$, $D_1(r) = D_2(r)$ and $D_3 = D_4$.

The state space form of the controller can be represented as:

$$\begin{aligned}
 \dot{x}_k(t) &= A_k x_k(t) + B_k y(t, r_L) \\
 u(t) &= C_k x_k(t) + D_k y(t, r_L)
 \end{aligned} \tag{5}$$

such that the closed loop system satisfies:

$$\inf_{K \in U} \sup_{w \in L_2[0, \infty)} J_\infty < \gamma^2 \quad (6)$$

where U is the set of all stabilizing controllers and γ is a constant.

The spatial cost function to be minimized as the design criterion of spatial H_∞ control design technique is:

$$J_\infty = \frac{\int_0^\infty \int_R y(t,r)^T Q(r) y(t,r) dr dt}{\int_0^\infty w(t)^T w(t) dt} \quad (7)$$

where $Q(r)$ is a spatial weighting function that designates the region over which the effect of the disturbance is to be reduced and J_∞ can be considered as the ratio of the spatial energy of the system output to that of the disturbance signal. The control problem is depicted in Fig. 3.

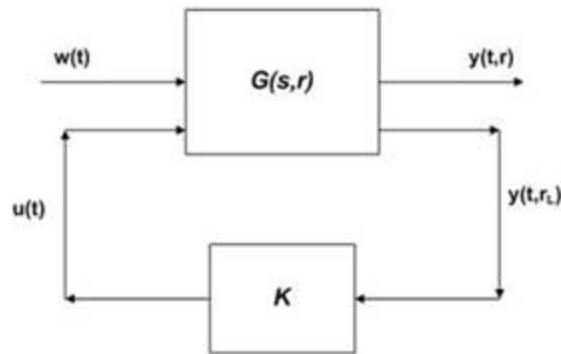


Fig. 3: The spatial H_∞ control problem of the smart beam

The spatial H_∞ control problem can be solved by the equivalent ordinary H_∞ problem [20] by taking:

$$\int_0^\infty \int_R y(t,r)^T Q(r) y(t,r) dr dt = \int_0^\infty \tilde{y}(t)^T \tilde{y}(t) dt \quad (8)$$

Hence, following the necessary mathematical manipulations, the state space representation given in equation (4) can be adapted for the smart beam model for a standard H_∞ control design as:

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\
\tilde{y}(t) &= \begin{bmatrix} \Pi \\ 0 \end{bmatrix} x(t) + \begin{bmatrix} \Theta_1 \\ 0 \end{bmatrix} w(t) + \begin{bmatrix} \Theta_2 \\ \kappa \end{bmatrix} u(t) \\
y(t, r_L) &= C_2 x(t) + D_3 w(t) + D_4 u(t)
\end{aligned} \tag{9}$$

The derivation of equation (9) and the below state space variables can be found in [19] as:

$$\begin{aligned}
A &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_1^2 & 0 & -2\xi_1\omega_1 & 0 \\ 0 & -\omega_2^2 & 0 & -2\xi_2\omega_2 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ \bar{P}_1 \\ \bar{P}_2 \end{bmatrix}, C_1 = \begin{bmatrix} \phi_1(r) \\ \phi_2(r) \\ 0 \\ 0 \end{bmatrix}^T, C_2 = \begin{bmatrix} \phi_1(r_L) \\ \phi_2(r_L) \\ 0 \\ 0 \end{bmatrix}^T \\
D_1 = D_2 &= \sum_{i=3}^{50} \phi_i(r) k_i^{opt}, D_3 = D_4 = \sum_{i=3}^{50} \phi_i(r_L) k_i^{opt}, \Pi = \begin{bmatrix} L_b^{3/2} & 0 & 0 & 0 \\ 0 & L_b^{3/2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Theta_1 = \Theta_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \left(\sum_{i=3}^{50} L_b^3 (k_i^{opt})^2 \right)^{1/2} \end{bmatrix}
\end{aligned}$$

One should note that, the control weight, κ , is added to the system in order to limit the controller gain and avoid actuator saturation problem. In the absence of the control weight, the major problem of designing an H_∞ controller for the system given in equation (4) is that, such a design will result in a controller with an infinitely large gain [14]. In order to overcome this problem, an appropriate control weight, which is determined by the designer, should be added to the system. Since the smaller κ will result in higher vibration suppression but larger controller gain, it should be determined optimally such that not only the gain of the controller does not cause implementation difficulties but also the suppressions of the vibration levels are satisfactory. In this study, κ was decided to be taken as 7.87×10^{-7} . The simulation of the effect of the controller is shown in Fig. 4 as a Bode plot, and the frequency domain simulation is done by Matlab v6.5.

The vibration attenuation levels at the first two flexural resonance frequencies were found to be 27.2 dB and 23.1 dB, respectively. The simulated results show that the designed controller is effective on the suppression of undesired vibration levels.

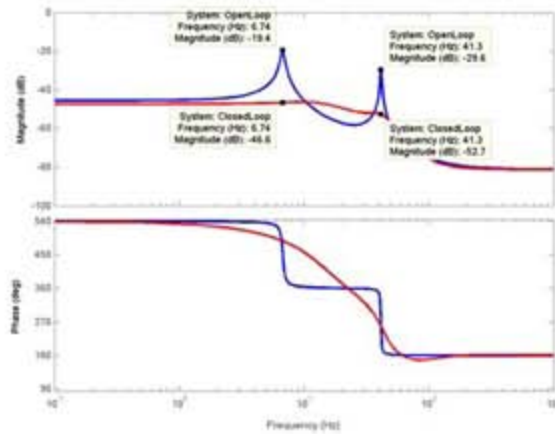


Fig. 4: Bode plots of the open and closed loop frequency responses of the smart beam under the effect of spatial H_{∞} controller

Experimental Implementation:

The smart beam of this study, shown in Fig. 5, consists of the PZT patches that are placed in a collocated manner to have opposite polarity and used as the actuators. A Keyence LB-1201(W) LB-300 laser displacement sensor (LDS) is used as the sensor.



Fig. 5: The smart beam used in the study

The closed loop experimental setup is shown in Fig 6. The displacement of the smart beam at location $r_L = 0.99L_b$ was measured by using the LDS and converted to a voltage output that was sent to the SensorTech SS10 controller unit via the connector block. The controller output was converted to the analog signal and amplified 30 times by SensorTech SA10 high voltage power amplifier before applied to the piezoelectric patches. The controller unit is hosted by a Linux machine, on which a shared disk drive is present to store the input/output data and the C programming language based executable code that is used for real-time signal processing.

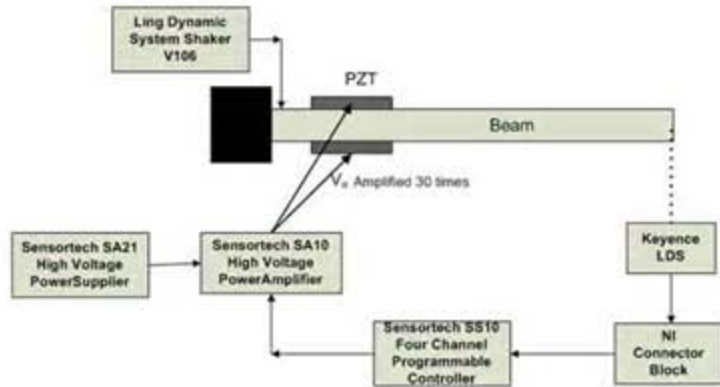


Fig. 6: The closed loop experimental setup

Free Vibration Suppression:

For the free vibration control, the smart beam was given an initial 5 cm tip deflection and the open loop and closed loop time responses of the smart beam were measured. The results are presented in Fig. 7.

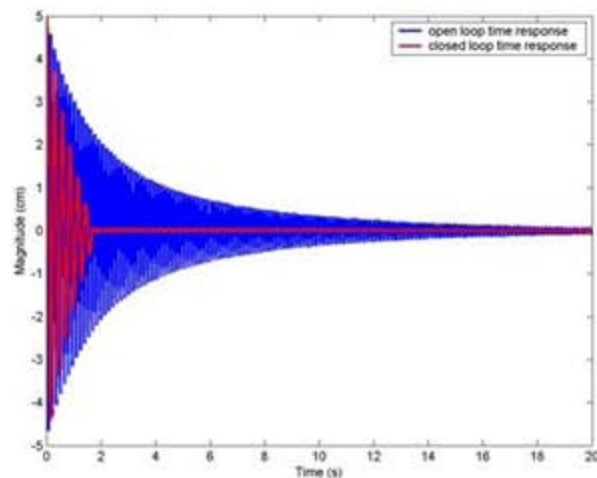


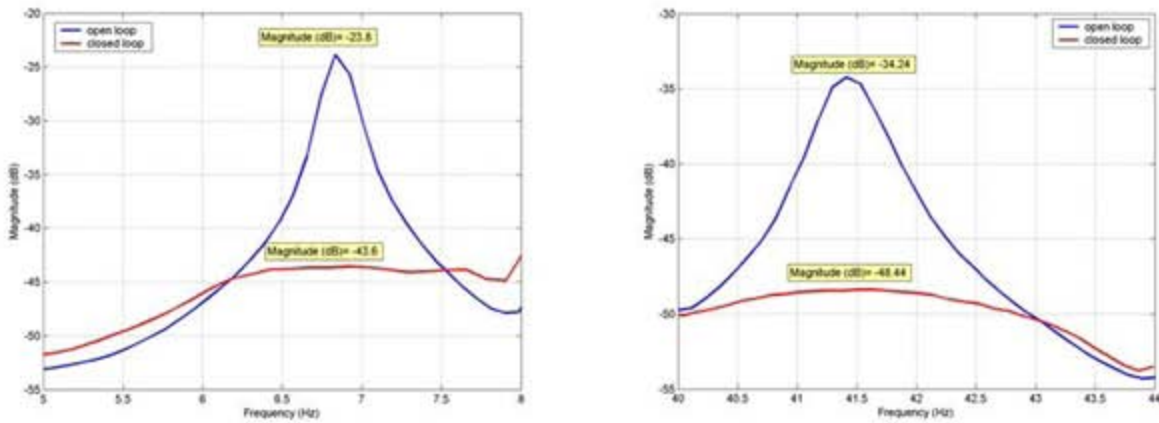
Fig. 7: Free vibration suppression of the smart beam

Fig. 7 shows that the controlled time response of the smart beam settles nearly in 1.7 seconds. Hence, the designed controller proves to be very effective on suppressing the free vibration of the smart beam.

Forced Vibration Suppression:

The forced vibration control of the smart beam was analyzed in two different configurations. In the first one, the smart beam was excited for 180 seconds with a mechanical shaker located very close to the root of the smart beam, on which a sinusoidal chirp signal of amplitude 4.5V was applied. The excitation bandwidth was taken first 5 to 8 Hz and later 40 to 44 Hz to include the first two flexural resonance frequencies separately. The experimental attenuation of vibration levels at first two resonance frequencies were

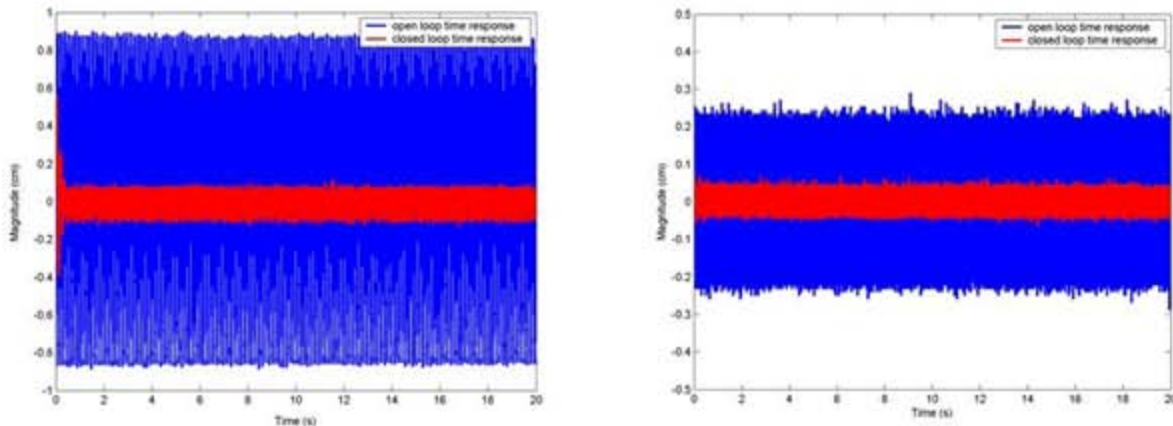
determined from the Bode magnitude plots of the frequency responses of the smart beam and shown in Fig. 8.a-b. The resultant attenuation levels were found as 19.8 dB and 14.2 dB, respectively. In the second configuration, instead of using a sinusoidal chirp signal, constant excitation was applied for 20 seconds at the resonance frequencies with a mechanical shaker. The ratios of the maximum time responses of the open and closed loop systems, shown in Fig. 9.a-b, are considered as absolute attenuation levels. Hence, for this case, the attenuation levels at each resonance frequency were calculated approximately as 10.4 and 4.17, respectively. Consequently, the experimental results show that the controller is effective on suppression of the forced vibration levels of the smart beam.



a) 5-8 Hz

b) 40-44 Hz

Fig. 8: Bode magnitude plot of the frequency responses of the open and closed loop systems under the effect of spatial H_{∞} controller



a) 1st resonance

b) 2nd resonance

Fig. 9: Open and closed loop time responses of the smart beam under constant excitation at resonance frequencies under the effect of spatial H_{∞} controller

Efficiency of the Controller:

The efficiency of spatial controller in minimizing the overall vibration over the smart beam was compared by a pointwise controller that is designed to minimize the vibrations only at point $r_L = 0.99L_b$. The implementations of the controllers showed that both pointwise and spatial H_∞ controllers managed to reduce the vibration levels of the smart beam due to its first two flexural modes in nearly equal significance [19]. The simulated H_∞ norms of the smart beam as a function of r , shown in Fig. 12., showed that spatial H_∞ controller has a slight superiority on suppressing the vibration levels over entire beam.

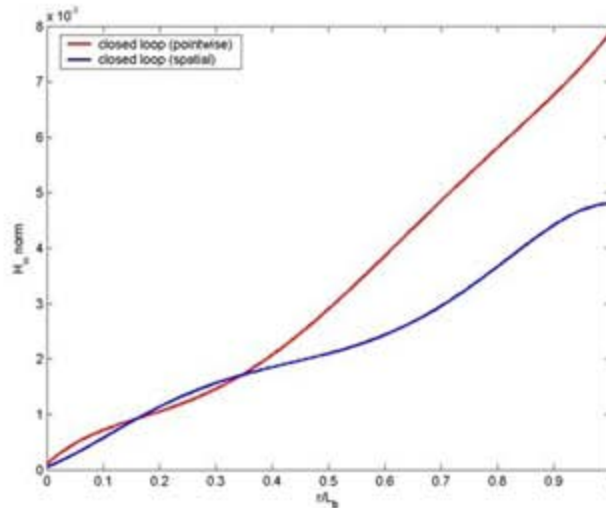


Fig. 10: Simulated H_∞ norm plots of closed loop systems under the effect of pointwise and spatial H_∞ controllers

CONCLUSION

This study presented the active vibration control of a cantilevered smart beam. A spatial H_∞ controller was designed for suppressing the first two flexural vibrations of the smart beam. The efficiency of the controller was demonstrated both by simulation and experimental implementations. The effectiveness of the spatial H_∞ controller on suppressing the vibrations of the smart beam over its entire body was also compared by a pointwise one. The results proved that, although spatial H_∞ controller has a slight superiority on suppressing the vibration levels over entire beam, a pointwise controller can also achieve successful attenuation of vibration levels as long as the design criteria of it is to suppress the vibrations at the point very close to the free end of the beam.

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